

Overlap Probabilities and Delay Detonators

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Teaching;
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Summary

A problem in mining engineering concerning detonators for rock blasting is considered. Simple probability calculations give very useful information about the detonation sequence.

◆ INTRODUCTION ◆

IT IS DIFFICULT to find fresh examples that motivate the variance formula for linear combinations of independent random variables. A consulting problem concerning delay detonators has proved useful in illustrating the formula and has been used in a variety of ways. It also has the added advantage of illustrating how even elementary techniques can be useful in “real life”.

◆ OVERLAP PROBABILITIES FOR BLAST DESIGNS ◆

A blast design gives a configuration of blastholes and detonators. The particular blast design shown in figure 1 is used for blasting weathered overburden using large diameter blastholes and is designed to fragment the rock and throw it forward in order to minimise rehandling. To maximise the rock breaking efficiency of the blast the holes are detonated “row by row”. This is achieved by using delay detonators which introduce a delay in the blast sequence. It is possible, however, for blastholes to detonate out of sequence because of the natural variability in the delay times. This is called an overlap. One of the objectives of the mining engineer is to minimise the probability of overlap by taking into account the variability of the delays in the design of the blast.

In the example blast design given in figure 1, the first row has a series of delay detonators with a nominal delay time of say 25 ms. Between each row there are delay detonators with nominal delays of say 100 ms. Finally in each of the blastholes are down lines with in-hole delay detonators nominally of say 400 ms. The detonators are connected together by detonating cord that can be considered infinitely fast.

The delay times of the detonators in the first row are labelled X_i . The delay times between subsequent rows are labelled Y_i , and the in-hole delay times are labelled Z_i . It is assumed that

$$X_i \sim N(\mu_x, \sigma_x^2)$$

$$Y_i \sim N(\mu_y, \sigma_y^2)$$

$$Z_i \sim N(\mu_z, \sigma_z^2)$$

and that all the delay times are mutually independent.

If T_{15} is the time of initiation of the blasthole labelled Z_{15} , for example, then

$$T_{15} = X_3 + X_2 + Y_2 + Y_8 + Z_{15}$$

Estimates of the parameters are available from quality control tests that are regularly performed.

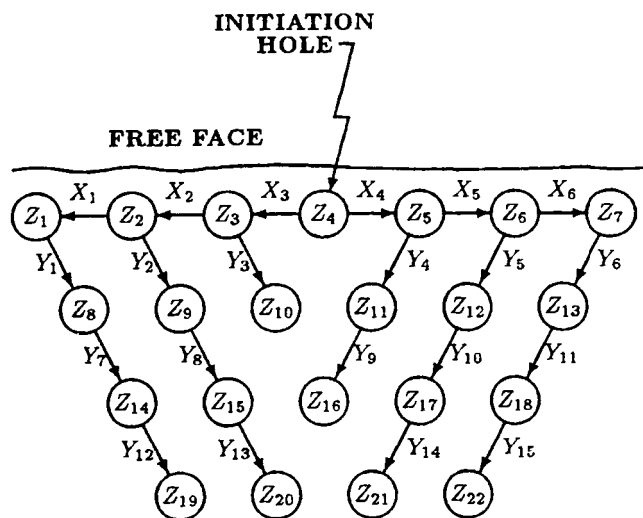


Fig 1. A plan view of a blast design

◆ CALCULATION OF OVERLAP PROBABILITIES ◆

Although it is strictly possible for holes that are not neighbours to each other to give rise to an overlap, in practice examination is usually confined to those neighbouring blastholes where out of sequence firing is more likely such as blastholes labelled Z_{14} , Z_{15} and Z_{19} ; and Z_{17} , Z_{18} and Z_{22} . If U denotes the time delay between blastholes labeled Z_{22} and Z_{18} , and V denotes the time delay between blastholes labelled Z_{22} and Z_{17} the probability of overlap is

$$P(U \leq 0 \text{ or } V \leq 0)$$

where

$$U = Y_{15} + Z_{22} - Z_{18}$$

$$V = X_6 + Y_6 + Y_{11} + Y_{15} + Z_{22} - Y_5 - Y_{10} - Z_{17}$$

and therefore U and V are jointly Normally distributed with

$$\mu_U = \mu_V$$

$$\mu_V = \mu_X + \mu_Y$$

$$\sigma_U^2 = \sigma_Y^2 + 2\sigma_Z^2$$

$$\sigma_V^2 = \sigma_X^2 + 5\sigma_Y^2 + 2\sigma_Z^2$$

Now

$$P(U \leq 0 \text{ or } V \leq 0) = P(U \leq 0) + P(V \leq 0) - P(U \leq 0, V \leq 0)$$

Calculation of $P(U \leq 0)$ and $P(V \leq 0)$ is done using Normal distribution calculations while calculation of $P(U \leq 0, V \leq 0)$ can either be done using specialised tables (Owen, 1956), calculated using series approximations (Daley, 1974) or set to zero to give an upper bound for the overlap probability.

◆ CALCULATION OF $P(U \leq 0, V \leq 0)$ ◆

To calculate $P(U \leq 0, V \leq 0)$, the covariance of U and V must be computed. This is a good opportunity to introduce the very useful formula

$$\text{Cov}(U, V) = \frac{1}{4} [\text{Var}(U + V) - \text{Var}(U - V)]$$

where for this blast design

$$U + V = X_6 + Y_6 + Y_{11} + 2Y_{15} + 2Z_{22} - Y_5 - Y_{10} - Z_{17} - Z_{18}$$

$$U - V = Y_5 + Y_{10} + Z_{17} - X_6 - Y_6 - Y_{11} - Z_{18}$$

giving

$$\text{Var}(U+V) = \sigma_X^2 + 8\sigma_Y^2 + 6\sigma_Z^2$$

$$\text{Var}(U-V) = \sigma_X^2 + 4\sigma_Y^2 + 2\sigma_Z^2$$

and hence

$$\text{Cov}(U, V) = \sigma_Y^2 + \sigma_Z^2$$

The same result could, of course, be obtained using the usual formula for covariances of linear combinations if desired.

◆ EXAMPLE ◆

Assume

$$\begin{array}{ll} \mu_X = 40 & \sigma_X = 4 \\ \mu_Y = 100 & \sigma_Y = 10 \\ \mu_Z = 400 & \sigma_Z = 40 \end{array}$$

then for this blast design

$$\begin{array}{ll} \mu_U = 100 & \sigma_U^2 = 3300 \\ \mu_V = 140 & \sigma_V^2 = 3716 \\ \text{Corr}(U, V) = 0.4855 \end{array}$$

From Normal distribution calculations

$$\begin{array}{l} P(U \leq 0) = 0.04086 \\ P(V \leq 0) = 0.01082 \end{array}$$

and therefore an upper bound for the probability of overlap is 0.05168. A more accurate value is obtained by calculating $P(U \leq 0, V \leq 0) = 0.00327$ using one of the series approximations given by Daley (1974). Hence the probability of overlap is 0.04841.

To get an upper bound for the probability of overlap for the entire blast, the analogous calculation could be repeated for each hole apart from those in the first row and the probabilities of overlap (or upper bounds) added.

◆ DISCUSSION ◆

This example has been used in a variety of ways. Sometimes I use it in lectures as an illustration of the variance formula. Other times I have used it as an examination question or for an assignment. It has also been used as a basis for a final year industry project.

Many extensions to the problem can be explored such as determining the probability of overlap for different types of blast designs or determining maximum variability for a specified probability of overlap. Students respond very positively to this interesting application of probability that shows the power of the use of only seemingly elementary methods.

References

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