The ‘Golden Egg’

KEYWORDS:
Golden Ratio,
Project; Investigation

Hilary Kimber
University of Surrey,
Guildford, England.

.

Summary
We examine a geometrical construction of a”perfect” egg shape, comparing it with students’ conceptions of the ideal shape. Dimensions of birds’ eggs are investigated, to find out how egg shapes are related to natural circumstances.

◆INTRODUCTION◆

WHILST I was browsing through a needlework magazine recently, I was interested to find a geometrical construction which produced a ‘perfect’ egg shape for use in Easter craft projects. I have reproduced the construction in the Appendix and from this it is easy to calculate the ratio of height to maximum width as 1.2929:1.

This set off two different trains of thought. Firstly, whether there is an aesthetically pleasing egg shape, with ‘golden egg’ ratio 1.2929 :1, akin to the Golden Ratio favoured by the ancient Greeks which is seen to great effect in the design of the Parthenon and other ancient buildings. Secondly, I began to investigate the height/width ratios of birds’ eggs, whose proportions are more a result of function than artistic perfection.

The investigations which I carried out are really only the tip of the iceberg: there is much more which could be done in the classroom, either as short statistical investigations or longer projects.

◆EGGS AND RECTANGLES◆

In order to investigate whether either of the ratios (egg or rectangle) was really ‘golden’, in the sense of being an aesthetically perfect example of its own type, I asked a group of 36 University students to draw ‘attractive’ rectangle and egg shapes on plain A4 paper. The joker of the group produced rather strange shapes (a tiny rectangle in one corner of the paper with a ratio of 8.4, and a similar very tiny elongated egg), so I discarded these, giving a final sample size of 35.

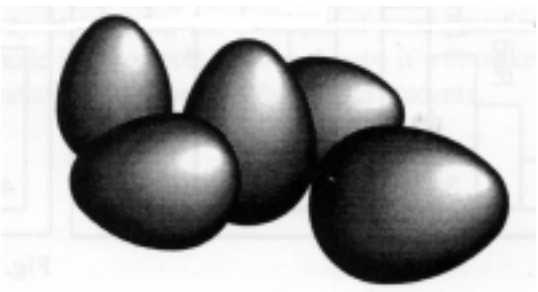
Summary values are given in Table 1.

Table 1. Summary of the drawn rectangle and egg dimensions.

	Rectangle ratio	**Egg ratio**
Sample size, n	35	35
Maximum	3.63	2.08
Minimum	1.22	1.09
Mean	2.14	1.50
s.e. of mean	0.14	0.03
95% conf. int	1.93 - 2.35	1.43 - 1.56

We can see that the ‘perfect’ values of 1.618 (rectangle) and 1.2929 (egg) are not included in the respective 95% confidence intervals, i.e. the students drew more elongated shapes than we might have hoped. In order to see whether the shape of paper influenced the shapes which were drawn, I handed out blank paper squares and repeated the experiment. However, the results were very close to those above, and this did not appear to be an influencing factor. So, although we may be disappointed that the ‘golden egg’ is not perceived to be so ideal, it seems that the golden rectangle is also out of fashion!

This investigation could be repeated with different groups of students, and in ways which would allow comparative tests to be carried out. Do girls’ and boys’ perceptions differ significantly (*t* - tests)? Do perceptions differ with age (analysis of variance on age categories)? The investigation itself could also be approached in different ways: by presenting half a dozen egg or rectangle shapes ready drawn and asking for orders of preference on aesthetic grounds,



one could obtain ranked data suitable for non-parametric analyses. In fact the shapes themselves could also be ranked in ratio order.

◆INVESTIGATION◆ OF BIRDS' EGGS

The shape of birds' eggs is a matter of natural convenience rather than aesthetic considerations. Given that the overall shape should be smooth to assist laying, Hoeher (1974) identifies several different basic egg shapes. From the point of view of efficiency, a spherical egg will maximise volume whilst minimising the amount of expensive shell required. Owls and birds of prey such as Eagles, Kestrels, Merlins and Peregrines lay 'short ovate' eggs, which are nearly spherical. This shape is fine for birds which nest in holes or build deep nests, as the eggs are unlikely to roll out of the nest. At the other extreme, however, many seabirds which nest on precarious ledges lay 'long pyriform' eggs, which are sharply tapered at one end. This allows the eggs to settle together, pointed end down, and minimises the risk of them rolling off the ledge.

However, leaving detailed considerations of shape aside, Hoeher lists typical heights and widths (mm) of the eggs of many species. I collected a list of 281 birds together with their egg dimensions and entered them into the spreadsheet Excel for analysis.

Figure 1 shows a scatter plot of egg height against egg width for the 281 eggs, on which I have superimposed the 'golden egg' ratio line y = 1 .2929x.

The plot shows that, as might have been expected, the majority of birds' eggs are rather 'thinner' than the 'golden egg'. In fact, only 44 of the listed species laid eggs whose declared ratio was less than 1.2929, and many of these were the birds of prey and owls previously mentioned.

However, the scatter diagram does display a rough proportionality between height and width (correlation = 0.988), with some heteroscedasticity: as might be expected, larger eggs show more variability in ratio (could this also partly be a function of the accuracy of measurement, all dimensions being given to the nearest 0.1 mm?) Also, one must bear in mind that the dimensions given are 'typical' egg sizes and that sizes within a species are not uniform. The least squares regression line for the 281 data points is

$$height = -1.41 + 1.43 width,$$

whereas that forced through the origin is

$$height = 1.39\ width.$$

Thus, for birds' eggs the average ratio would appear to be around 1.4.

Table 2 shows some numerical summary values for the heights, widths and ratios of the 281 eggs.

Table 2. Summary dimensions of the birds' eggs.

	Height (mm)	**Width (mm)**	**Height/Width**
Maximum	115.00	74.10	1.640
Minimum	13.50	10.30	1.160
Mean	37.30	27.10	1.370
s.e. of mean	1.14	0.79	0.005

We see that the ratios varied from 1.16 (the Scops Owl egg) to 1.64 (the Guillemot and Shag eggs), with a mean of 1.37. To hark back to our 'golden egg' ratio, the egg closest to being 'perfect' had a ratio of 1.2941, so we might conclude that the Blue Tit, rather than the Goose, is the bird which lays the golden egg!

There is considerable scope within the data set for contrasting ratios from different classes of birds. Not being an expert in ornithology and without detailed knowledge of nesting sites, etc., I have chosen, as an example, three easily identified groups within the data: the owls, birds of prey and swans/ducks/geese. Table 3 shows summaries of the height/width ratio for these three

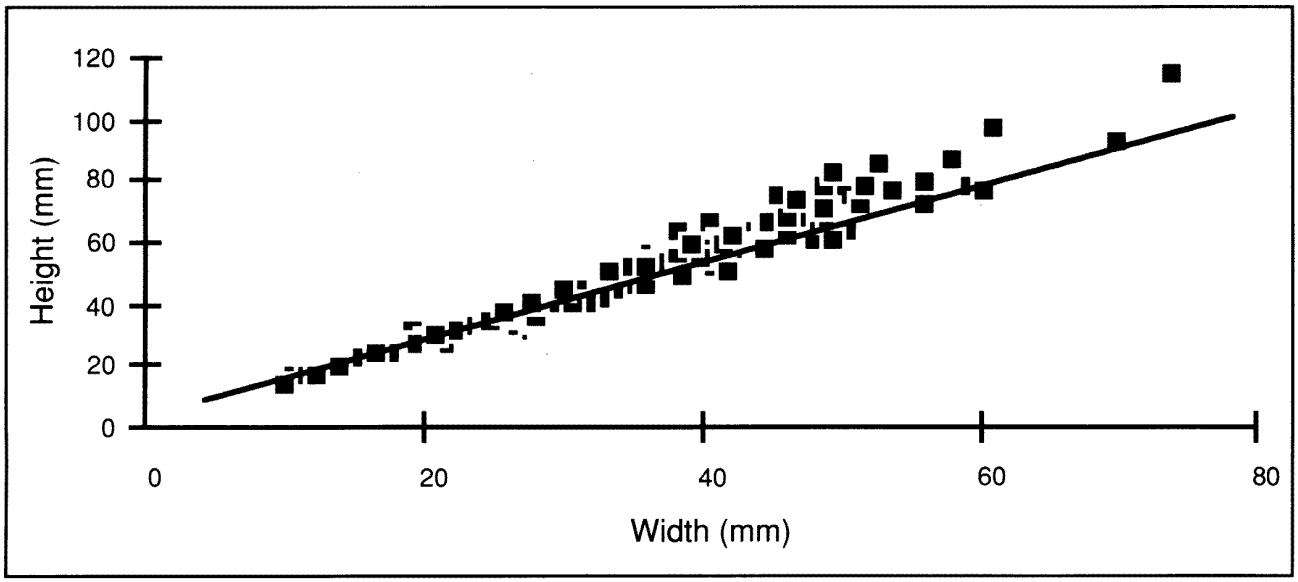


Fig. 1. Dimensions of 281 birds' eggs, with the 'golden egg' line $y = 1.2929x$.

different types.

We see that the mean ratio for the swans, ducks and geese differs quite a lot from those of each of the other two groups. Is this connected with the fact that the first two groups are land birds whereas the third are water birds? Can we identify other groups to confirm this difference, and if so, can we say why they should differ?

Table 3. Comparison of ratios for eggs of different types of birds.

	Owls	**Birds of prey**	**Swans, ducks, geese**
Sample size	10	15	21
Maximum ratio	1.273	1.328	1.552
Minimum ratio	1.159	1.236	1.352
Mean ratio	1.226	1.273	1.426
s.e. of mean ratio	0.012	0.0073	0.012

There are many more comparisons which could be made in a similar way using the lists in Hoeher. Do eggs from different types of nesting sites (tree, hole or ledge) differ significantly in shape? Do closely related birds have similarly shaped eggs, or are there differences due to different nesting sites within a related group? There is much scope here for both descriptive statistics (numerical summaries, scatter plots using different symbols for different types of bird) and comparative testing.

Hoeher also records information about other variables for all but a few of the listed birds. Egg weight, normal clutch sizes and typical incubation times are recorded for the vast majority of the birds. Do larger/heavier eggs take longer to incubate and if so, what is the nature of the relationship? Does the number of eggs in a clutch affect incubation time? One could even compare some of the egg sizes with nest sizes, as these are also recorded for some of the birds. Do birds which lay near-spherical eggs build deeper nests?

Figure 2 shows a scatter plot of egg weight versus egg height; and shows that, although there are still signs of heteroscedasticity as you might expect, there is evidence of some relationship between these variables. Would it be possible to find a function of height and width which shows a linear relationship with weight (perhaps by approximating the egg shape by a cylinder or sphere)? Obvious candidates here would be plotting weight against $(\text{height})^3$, or against $(\text{width})^2 \times \text{height}$.

We should not ignore the qualitative variables recorded when considering possible analyses: the nesting sites, egg colours and plain/speckled appearances are all useful categorical variables to work on. Do hole-nesting birds tend to lay darker eggs and ledge-nesting birds lighter eggs as camouflage? Tests of association may be useful here.

◆A 'HANDS-ON' APPROACH◆

As an alternative to reading data from a book, students may prefer to collect their own 'raw egg' (!) data. Two readily-available sources are hens' eggs and (especially around Easter time) chocolate eggs.

I carried out a rather cursory investigation into hens' eggs by measuring eight size 3 eggs (the current contents of my refrigerator) with calipers. The summary values for this rather small sample are shown in Table 4, but because of the uniformity of size of the graded eggs there is too little variability to be interesting.

Hens' eggs are very good for investigations, whether supermarket-bought or as raw data straight from the

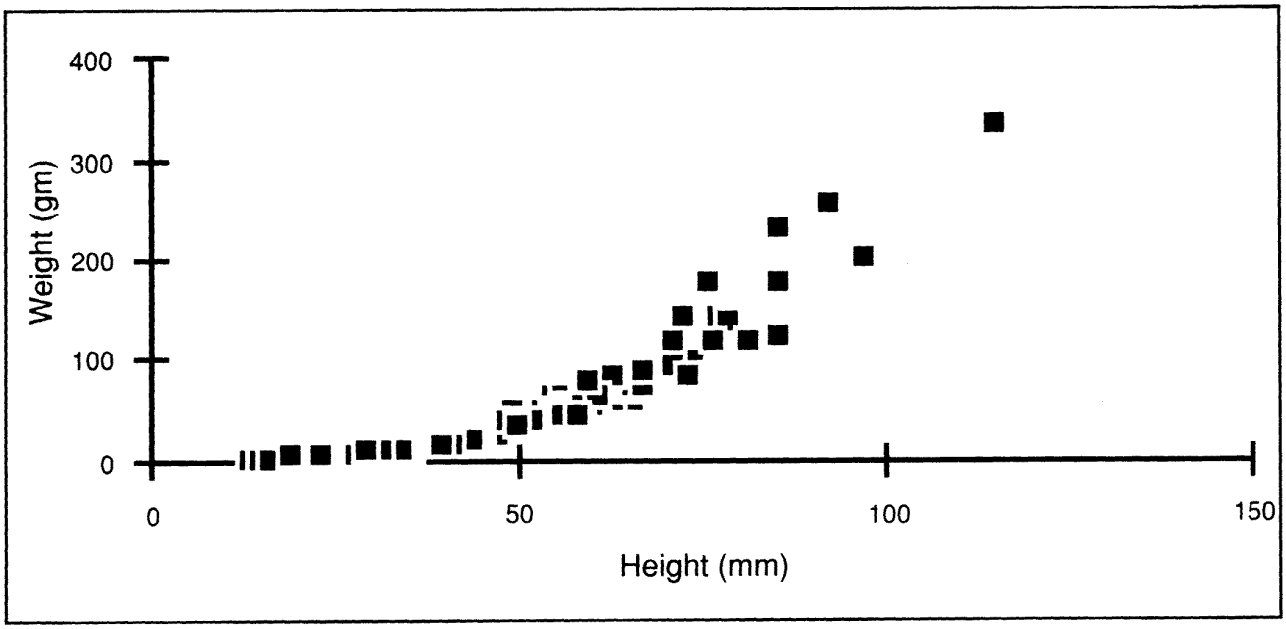


Fig. 2. Scatter plot of egg weights and heights.

birds. One could investigate the weight differences between different grades of shop-bought eggs and calculate the within-grade and between- grade variability (analysis of variance). Data from poultry farms or smaller egg producers hold many possibilities for analysis. Do different breeds of hens have differently shaped eggs? Are brown eggs a different shape from white eggs? How much does the height/width ratio vary in eggs from a single bird? Does the age of a bird affect the shape or weight of its eggs? Are eggs from matched pairs of free-range and battery hens substantially different in some way(s)?

Table 4. Summary of dimensions of the hens' eggs.

	Height (cm)	Width (cm)	Height/Width
Maximum	6.000	4.400	1.390
Minimum	5.600	4.200	1.270
Mean	5.780	4.330	1.340
s.e. of mean	0.049	0.025	0.016

Finally, the topic of chocolate eggs returns us to our starting point of the 'golden egg'. Chocolate eggs are made to be sold, so perhaps if manufactured with dimensions in an ideal ratio a golden chocolate egg would outstrip its competitors. At the time of writing the only eggs available were Cadburys Creme Eggs. Measurements of a small number of these revealed little variation (at least to the degree of accuracy attainable): the eggs are obviously made in a mould measuring 5cm. by 3.5 cm., thereby obtaining a non-golden ratio of 1.43. One could compare these dimensions with other small eggs and also measure Mini Eggs and the larger boxed eggs. Do different companies make eggs with different height/width ratios? What is the order of preference among half a dozen similar sized eggs, and are the preferred eggs closer to the golden egg shape? Such investigations have the advantage that one can enjoy destroying the data afterwards. Happy sampling!

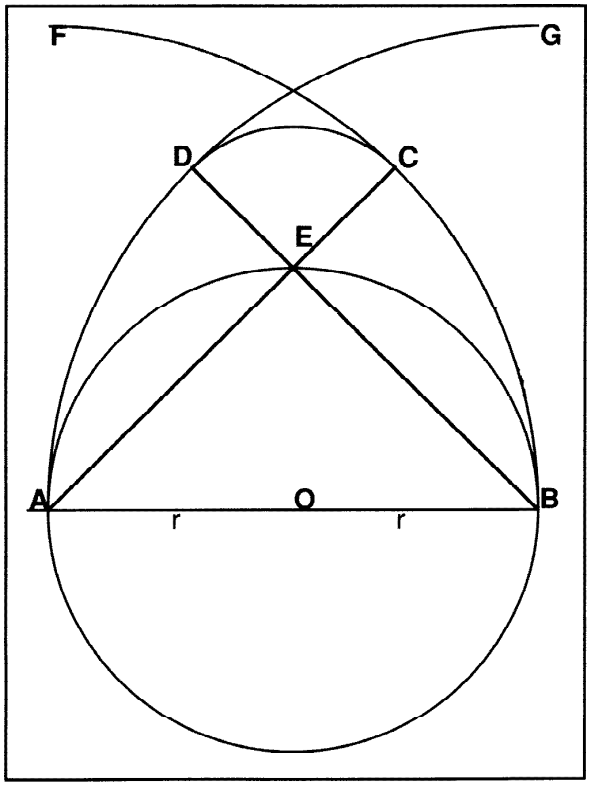


Fig. 3.

References and Further Reading

Harrison, C. (1975). *A Field Guide to the Nests, Eggs and Nestlings of British and European Birds.* Collins.

Hoeher, S. (adapted W. Reade) (1974). *Birds' Eggs and Nesting Habitats.* Blandford.

Acknowledgement

I am very grateful to the referee whose comments on the first draft of these investigations led me into a more ornithological frame of mind.

APPENDIX: The Construction of the Egg

The construction of the egg is shown in Figure 3. We first draw a circle of some radius *r,* and construct the isosceles triangle AEB whose base is the horizontal diameter of the circle. BF and AG are arcs of circles of radius *2r* with centres at A and B respectively. The lines AE and BE are extended to C and D respectively to meet these arcs. Lastly the circle quadrant CD is drawn with centre E to complete the shape.

A little elementary geometry reveals that the ratio of the egg's height to its maximum width is

$$4 - \sqrt{2}:2 \text{ or } 1.2929:1.$$

This 'golden egg' ratio is rather below the well-known golden ratio, which is

$$\frac{1+\sqrt{5}}{2}:1 \text{ or } 1.6180:1.$$

The completed egg shape is very similar to half an ellipse placed on top of a semicircle, although the upper portion is not quite elliptic (it is slightly too 'thin'). The difference is very small, but the above method has the advantage of being easy to construct with ruler and a pair of compasses.