

How Long will it take to Build Jack's House?

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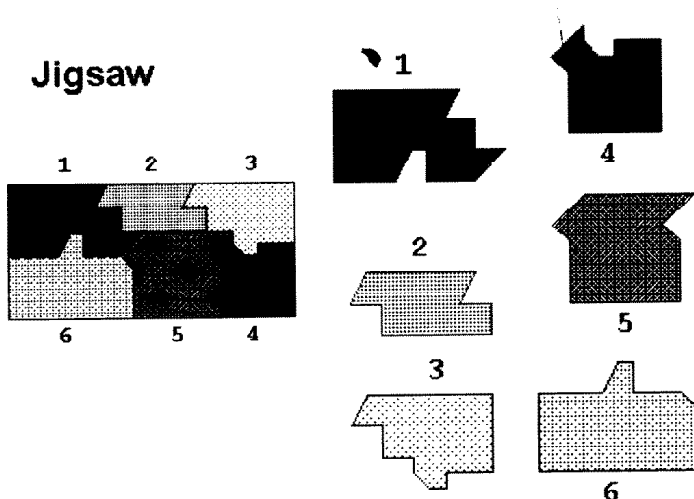
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◆INTRODUCTION◆

IN THIS article we describe two simulations undertaken by a class of 14 year old pupils as part of their Standard Grade mathematics class work. Both simulations refer to simple jigsaw games. Although the curriculum in Scottish schools does not include Probability or Statistics in a formal way at this level, these simulations provided opportunities to engage in stochastic thinking, while developing problem solving and number skills. We also include programs for computer simulation of the problems.

◆A COLLECTOR'S PROBLEM◆

The Collector's Problem has many guises. The following version was chosen as a suitable introduction to The House That Jack Built. It consists of a jigsaw with six pieces as shown below. Each piece is a different colour and shape, and it is a very simple jigsaw to put together. However, the rule is that you have to roll a die and you may only select the piece with the same number as is showing on the die. It does not matter in which order you collect the pieces. The question is, how many rolls of the die will be needed to complete the jigsaw?



◆SIMULATION◆

Preliminary class work involved exploring the output of a calculator's random number button, and converting these decimal numbers to whole numbers in the range 1 to M inclusive, using the formula: $INT(Ran\# \times M) + 1$. This was easily motivated, as the National Lottery had commenced the previous week, so that $M = 49$ was a popular choice for the multiplier. A graphics calculator which employs direct algebraic logic was used. This meant that pressing the execute button repeatedly gave a ready supply of random digits in the required range. A spreadsheet package would be another good source.

With the following week's lottery numbers safely noted, $M = 6$ was chosen, and rolls of a die were simulated. Each pupil contributed a tally of about 100 random digits to the class total (2300 rolls of a die!). Although any one pupil's contribution may show considerable variability, the resulting frequency table for the whole class showed remarkable regularity, each score occurring about 17% of the time. It was agreed that the calculator was a fair and convenient substitute for rolling a real die.

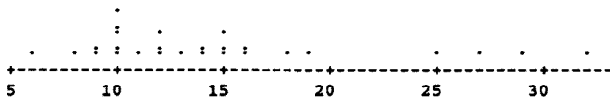
It is worth noting that about half the class did not correctly complete their tally at the first attempt. When tabulating 100 random digits in the range 1 to 6, the majority of pupils scanned the list and counted all the 1's, then all the 2's, and so on. Finally, to keep the teacher happy, they quickly entered the required number of tally marks to agree with the frequency they had counted earlier. The few pupils who used the correct method of recording tallies first, and then counting tallies, invariably completed their work correctly first time!

The pupils were now asked to use their newly discovered skills in simulation to answer the following question:

"How many rolls of the die will it take on average to complete the jigsaw?"

Many different answers were obtained by members of the class, as shown in the dotplot below:

Class Results for the Collector's Problem

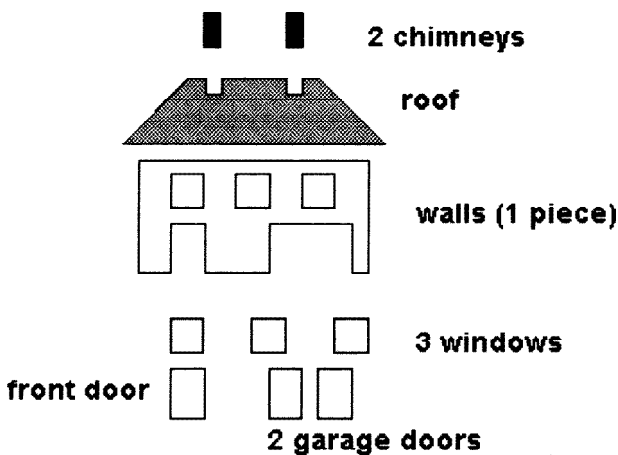


Using this data set, the pupils calculated the mean to be 14.7, a quite fortuitous result, which coincides with the theoretical answer, Green (1989a, b)! It was noted that the mean was not in the middle, there being more values below the mean than above it. The median, minimum, maximum, and range were calculated. One pupil completed her house in the absolute minimum of 6 turns.

◆JACK'S HOUSE◆

The *House That Jack Built* is a popular game for young children that is manufactured by I W Spear & Sons PLC. Each player must put together a simple jigsaw in the shape of a house. The jigsaw has 10 pieces as shown

The House That Jack Built



In the boxed game, each of the six types of piece is identified by a different colour. A die is provided whose sides are coloured to match the colours of the pieces. When you roll the die, you may only select one part of the house corresponding to the colour of the face that is showing. Once you have collected all the pieces of a particular colour, you do not add to your house on a future turn when you throw that colour again.

Unlike the simple jigsaw, the order in which you collect the pieces is also important:

1. you may not collect any other pieces until you have the walls (1 piece);

2. once you have the walls you may collect any of the other pieces except the chimneys;
3. you may only collect the chimneys once you have the walls and the roof.

Simulating this game is closer to real life applications, where complex conditioning is often an important feature. The numbers from 1 to 6 were chosen to correspond to the different parts of the house (not colours, as in the original game). The choice was arbitrary (it does not have to be a "6" to start!), but the mapping we used was:

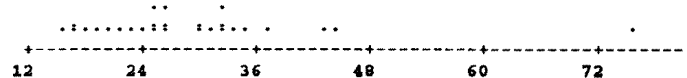
- 1 → Walls 2 → Front door 3 → Window
4 → Garage door 5 → Roof 6 → Chimney

The simulation is quite demanding, and requires pupils to develop efficient methods for recording progress according to the rules, while counting the number of throws of the die.

A check list such as the following provided a helpful prompt:

| Score | House part | Got it! ✓ |
|-------|--------------|--|
| 1 | Walls | <input type="checkbox"/> |
| 2 | Front door | <input type="checkbox"/> |
| 3 | Windows | <input type="checkbox"/> <input type="checkbox"/> <input type="checkbox"/> |
| 4 | Garage doors | <input type="checkbox"/> <input type="checkbox"/> |
| 5 | Roof | <input type="checkbox"/> |
| 6 | Chimneys | <input type="checkbox"/> <input type="checkbox"/> |

The dotplot below shows the class results



The mean was calculated to be 29.3. Again it was noted that the mean was not in the middle, there being more values below the mean than above it. On this occasion no one scored 10, the minimum possible number of rolls of the die. There was one very high result of 76.

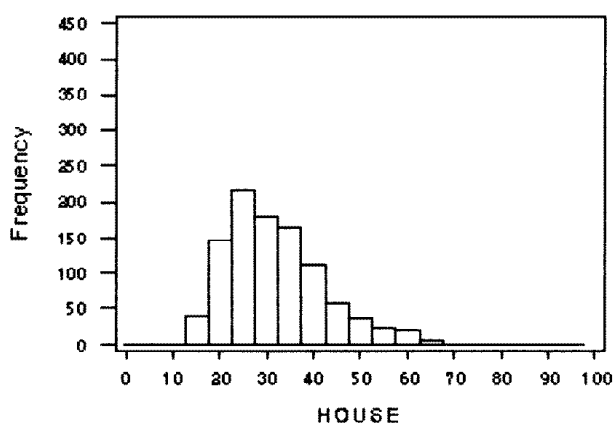
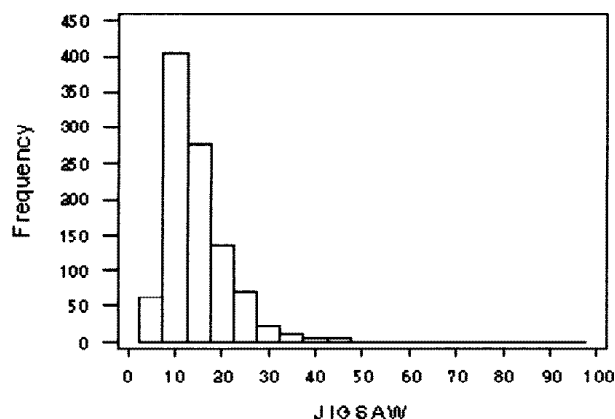
◆COMPUTER SIMULATION◆

Computer simulations can be used to extend the work the pupils themselves do in class. We have appended listings for two short computer programs in QBASIC. The results are saved as ASCII files for subsequent analysis. Summary statistics for 1000 repetitions of the simulations described above are shown below

Results of Computer Simulations

| | MEAN | MEDIAN | STDEV |
|--------|--------|--------|--------|
| JIGSAW | 14.707 | 13.000 | 6.785 |
| HOUSE | 31.771 | 30.000 | 10.388 |

| | MIN | MAX |
|--------|--------|--------|
| JIGSAW | 6.000 | 57.000 |
| HOUSE | 12.000 | 79.000 |



Computer simulation permits the above investigations to be extended in a number of ways. For example, depending on the level of pupil, additional activities might include

1. Investigate how many turns it would take to complete a simple jigsaw with a different number of pieces (for example 10 pieces), when there are no conditions on the order in which the pieces are collected.
2. Investigate how the order conditions affect the average number of turns required. What is the effect of having identical pieces of certain component parts (e.g. 3 windows, 2 garage doors, etc.) on the average number of turns required?

◆DISCUSSION◆

The classroom activities described here were completed in four 50 minute periods, and were greatly enjoyed by the whole class. The simulations produce

interesting data, which can be used to motivate various descriptive statistics. For example, when answering the question of interest, both median and mean can be discussed. However, the idea of “average” is only part of the story. These data display considerable variation and, depending on the pupils’ level, this might lead to a discussion of range, outliers, simple graphical representations such as dotplots and histograms, standard deviation, and so on. We have already noted that *The House That Jack Built* provides a good introduction to realistic simulations, because of the conditions involved. There are many other possibilities along the same lines. For example, in a Beetle Drive, each competitor must draw a beetle with one body, one head, two feelers, two eyes, six legs and one tail. The six faces of a die each correspond to one type of body part, and play proceeds much as in *The House that Jack Built*. The body must be drawn before any other part can be added, and the head must be drawn before the eyes or feelers. Simple games such as these, and certain aspects of more complicated games, are a rich source of problems which can be successfully solved by simulation. Simulation is a powerful way to give pupils access to interesting problems, even in those cases where an analytical solution is either unknown or beyond their level.

References

- Green, D, (1 989a). Problem Page. *Teaching Statistics* 11(2), 56.
 Green, D, (1989b). Problem Page. *Teaching Statistics* 11(3), 94-95.

Acknowledgements

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Computer Program 1: The Collector’s Problem

The results are stored in an ASCII file which can be read into any statistics or spreadsheet package for analysis. Line 180 can be edited to provide a different path name, or re-coded to permit a path name to be typed in when the program is running.

```
100 CLS : RANDOMIZE TIMER
110 PRINT "The Collector’s Problem": PRINT
120 PRINT "How many items are there in the full set
130 INPUT number%
140 DIM Item(number%)
150 PRINT: PRINT "How many repetitions of the
```

simulation

```
160 INPUT repet%
170 n$ = "A:\RESULT.DAT"
180 PRINT "saving answer to";n$
190 OPEN n$ FOR OUTPUT AS #1
200 FOR p% = 1 TO repet%
210 REDIM Item(number%)
220 i% = 0
230 DO
240 i% = i% + 1: Throw = INT(RND*number%)
+1
250 FOR j% = 1 TO number%
260 IF Throw = j% THEN Item(j%) = 1
270 NEXT
280 set% = 0
290 FOR k% = 1 TO number%
300 set% = set% + Item(k%)
310 NEXT k%
320 LOOP UNTIL set% = number%
330 PRINT #1, i%
340 NEXT p%
350 CLOSE: END
```

Computer Program 2: The House that Jack Built

```
100 CLS : RANDOMIZE TIMER
110 PRINT "The House that Jack built";: PRINT
120 PRINT "How many repetitions of the
simulation";
130 INPUT repet%
140 n$="A:\RESULT.DAT"
150 PRINT "saving answer to";n$
160 OPEN n$ FOR OUTPUT AS #1
170 FOR p%=1 TO repet%
180 frtdoor=0 : chmpots=0: windaes=0 : gardors=0
190 started=0 : roofon=0 : built=0 : count%=0
200 DO
210 count=count%+1 : die=INT(RND*6)+1
220 IF die= 1 THEN started =-1
230 IF die=2 AND started THEN frtdoor=frtdoor+1
240 IF die=3 AND started THEN
windaes=windaes+ 1
250 IF die=4 AND started THEN
gardors=gardors+1
260 IF die=5 AND started THEN roofon=-1
270 IF die=6 AND roofon THEN
chmpots=chmpots+1
280 IF frtdoor>0 AND windaes>2 AND gardors> 1
AND chmpots>1 THEN built=-1
290 LOOP UNTIL built
300 PRINT #1, count%
310 NEXT p%
320 CLOSE: END
```