Lotteral Thinking?

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◆THE PROBLEM◆

THE UK National Lottery has perhaps had one positive spin-off, that of making members of the public think about an area of mathematics, however casually and intuitively. People see the draw each week on television, and when they see the six numbers (disregarding the bonus ball), they wonder how likely some particular feature of the set of numbers is, or was. Of course such 'wondering' may be regarded as a dubious statistical practice; perhaps ideally one should think about the chances of outcomes *before* the draw However, it does seem natural for many people to regard some sets of numbers as striking or unusual, worthy of comment.

Amongst the outcomes noted are, for example, three or four numbers in one 'decade' such as the forties, a preponderance of even or odd, and the existence of at least one pair of adjacent numbers. It is this last feature, and a possible lesson to be learnt from its exploration, that forms the subject of this note.

◆THE SOLUTION◆

When I looked at this problem, I eventually solved it after much toil, involving a gradual build up of all the different ways for the complementary problem of having non-adjacent numbers. (Arguably with any combinatorial or probabilistic problem it should be second nature to consider at an early stage whether the complementary problem might be easier.) It is not my intention to describe the method here, I shall merely label this the *'long way'*.

The answer is in fact [44! x 43!]/[38! x 49!] which equals 0.5048. Thus, somewhat counter-intuitively perhaps, adjacent numbers will occur every other week on average, approximately. The presence of the factorials in the answer acts as a trigger, and a little juggling will convert the answer to ${}^{44}C_6/{}^{49}C_6$. This then should lead us to try to find a combinatorial interpretation. The denominator is of course the number of different possible lottery draws, so the question remains as to how the numerator is choosing 6 from ***44'.*** Why on earth 44?

The trick is to set up a one-to-one correspondence between the non-adjacent ways with 6 from 49 that we are interested in and **all** ways of choosing 6 from 44. We do this as follows, showing how to move from a way of the former to one of the latter, and vice-versa. As an example, consider one of our non-adjacent ways:

4 14 25 27 33 47.

We convert this to a way from 44 by subtracting respectively 0,1,2,3,4,5

4 13 23 24 29 42.

Note firstly that the last number here could be at most 44. Secondly, since there can be no adjacent numbers in the first row above, there is no danger that there will be any repetitions in the second row, in that we are subtracting just one more each time as we move across.

Next, note that the process is reversible. We start with any set of 6 from 44, and add respectively 0,1,2,3,4,5: this gives a last number at most 49, and will never give two numbers adjacent since we are adding one more each time as we move across.

We thus have the required one-to-one correspondence between the two types.

◆THE LESSON◆

This method of obtaining the answer is the *'short way'*. However it would not be honest to call it the *'easy way'*, since there is hindsight involved. One might claim that it would have been sensible to have looked for a short way originally, but could the existence of a simple expression have been anticipated? I think not. Although it is always worth thinking there may be a short solution to a problem, we may have to admit that, when one does exist, it may not be at all obvious how to find it!